Journal of Applied Mechanics and Technical Physics, Vol. 41, No. 1, 2000

EIGENVIBRATIONS OF A FLEXIBLE PLATFORM FLOATING ON SHALLOW WATER

L. A. Tkacheva

UDC 532.591

The spectral problem for a flexible platform (airport) floating on shallow water is examined. The platform is modeled by a flexible plate of finite width and infinite length. A waveguide eigenmode is detected which propagates along the platform and attenuates exponentially away from it. The remaining eigenmodes are outgoing and growing exponentially away from the platform. All the eigenmodes can be excited only by external action on the platform. The behavior of the platform under external loading is examined.

Recent interest in the problem of floating flexible platforms is connected with the use of platforms in shelf exploitation, designing floating airports, etc., [1, 2]. Of particular concern is the study of the spectral properties of the problem, which makes it possible to predict dangerous effects on the platform, leading to elevated amplitudes of vibration and stresses. It should be noted that in most studies, the immersion of the platform in water is ignored i.e., the boundary conditions on the undersurface of the platform are extended to the unperturbed water surface. The results of the present study demonstrate that the ratio of the depth of immersion of the platform in the liquid is a small but important parameter for the eigenvibrations of the platform on shallow water.

Formulation of the Problem and Method of Solution. The platform is modeled by a flexible plate of constant thickness, finite width, and infinite length. It is assumed that the liquid is ideally incompressible and its depth is small compared to the dimensions of the plate but is great compared to its thickness. Therefore, the liquid flow can be described using shallow-water theory. The vibration amplitudes are assumed to be small, and the problem is considered in a linear formulation.

According to shallow-water theory [3], the liquid-velocity potential φ on the free part of the liquid surface satisfies the wave equation

$$\Delta\varphi - \frac{1}{gH}\frac{\partial^2\varphi}{\partial t^2} = 0, \tag{1}$$

where g is the acceleration of gravity, H is the liquid depth, and t is time. On the undersurface of the plate, the following relations hold [3]:

$$w_t = -(H - d)\Delta\varphi; \tag{2}$$

$$p = -\rho\varphi_t - \rho g(w - d). \tag{3}$$

Here w are the normal displacements of the plate, d is the of immersion of the plate in water, p is the pressure on the undersurface of the plate, and ρ is the liquid density.

The vibration of the plate is described by the equation [4]

$$D\Delta\Delta w + \rho_0 h\frac{\partial^2 w}{\partial t^2} = p. \tag{4}$$

Lavrent'ev Institute of Hydrodynamics, Russian Academy of Sciences, Novosibirsk 630090. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 41, No. 1, pp. 173–181, January–February, 2000. Original article submitted August 24, 1998.

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Here D is the cylindrical rigidity of the platform, h is its thickness, ρ_0 is the density of the material, and Δ is the two-dimensional Laplace operator with respect to the horizontal coordinates.

The edges of the plate are not fixed. Since the plate is partially immersed in water, the liquid exerts a force on the plate edges. According to the laws of mechanics, the force acting on a system is equal to the change in the momentum of the system in unit time. We introduce Cartesian coordinates with the y axis directed along the plate, the x axis directed in the transverse direction, and the z axis directed in the vertical direction. The plate of width l occupies the segment $-l/2 \leqslant x \leqslant l/2$. We examine the liquid region located between the cross sections $x = x_1$ and $x = x_2$ ($x_1 < l/2 < x_2$). The change in momentum in unit time is equal to the difference of momentum fluxes through the cross sections $x = x_1$ and $x = x_2$: $\rho(H - d)(\varphi_x)^2\Big|_{x=x_1} - \rho H(\varphi_x)^2\Big|_{x=x_2}$. This quantity is a second-order small parameter with respect to the vibration amplitude. In the linear approximation, the transverse force and the moment at the plate edges are set equal to zero:

$$\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} = 0, \qquad \frac{\partial}{\partial x}\Big(\frac{\partial^2 w}{\partial x^2} + (2 - \nu)\frac{\partial^2 w}{\partial y^2}\Big) = 0. \tag{5}$$

(ν is Poisson's constant).

A solution that is periodic in time is sought in the form $\varphi = \sqrt{gH}\, l\Phi \exp(-i\omega t)$, and the space variables are related to the characteristic horizontal dimension l. Then, from (1), for the amplitude of the potential Φ, we obtain the Helmholtz equation on the free part of the liquid surface:

$$\Delta\Phi + \lambda^2\Phi = 0. \tag{6}$$

Here $\lambda = \omega l/\sqrt{gH}$ is the dimensionless complex frequency. In this case, radiation conditions implying the absence of arriving waves should be satisfied at infinity to the left and to the right of the plate.

Substituting (2) and (3) into (4), we write the following equation for the liquid-velocity potential under the plate:

$$\Delta^3\Phi + (G - B\lambda^2)\Delta\Phi + G\lambda^2\Phi/(1 - \varepsilon) = 0. \tag{7}$$

Here $G = \rho g l^4/D$, $B = \rho_0 g h H l^2/D$, and $\varepsilon = d/H$ are dimensionless input parameters. The assumptions made in the formulation of the problem imply that $B \ll G$ and $\varepsilon \ll 1$. In the derivation of Eq. (7), the weight of the plate and the hydrostatic pressure component are ignored since they influence only the static part of the plate deflection.

Besides conditions (5) at the edges of the plate, the conjugation conditions on the boundary between the two regions should be satisfied. The conjugation conditions are deduced in [3] from the laws of conservation of mass and energy. At the edges of the plate, the potential is continuous, and the normal derivative has a discontinuity. For the plate, these conditions take the form

$$\begin{aligned} &\Phi(l/2 - 0) = \Phi(l/2 + 0), \qquad (H - d)\Phi_x(l/2 - 0) = H\Phi_x(l/2 + 0), \\ &\Phi(-l/2 - 0) = \Phi(-l/2 + 0), \qquad (H - d)\Phi_x(-l/2 + 0) = H\Phi_x(-l/2 - 0). \end{aligned} \tag{8}$$

The problem is solved by the following method. A general solution with unknown coefficients in each of the regions is constructed. From the boundary conditions and the conjugation conditions, we obtain a system of linear algebraic equations for the unknowns coefficients. The existence condition for a nontrivial solution is the equation to zero of the determinant of this system. Investigation of the eigenvibrations of the plate on water reduces to seeking values of the spectral parameter λ for which the determinant of the system is equal to zero. These values are found numerically using the principle of argument [5].

Absolutely Rigid Platform. We consider an absolutely rigid plate and the rigid eigenvibrations of the plate on water. Its normal displacements are representable as $w = l(\alpha_0 x + z_0)\exp(-i\omega t)$, where z_0 and α_0 are the amplitudes of vertical displacement of the center of mass and rotation around it. The displacements of the plate are related to the potential and pressure by relations (2) and (3).

In (2) and (3), we convert to the dimensionless variables P and Φ using the formulas

$$p = \rho g l P \exp(-i\omega t), \qquad \varphi = \sqrt{gH}\, l\Phi \exp(-i\omega t),$$

and relate the coordinate x to the characteristic dimension l. As a result, we obtain

$$\frac{\partial^2 \Phi}{\partial x^2} = i\lambda \frac{l}{H-d}(z_0 + \alpha_0 x), \qquad P = i\lambda \frac{H}{l}\Phi - \alpha_0 x - z_0,$$

Hence, the potential and pressure under the plate are

$$\Phi = i\lambda \frac{l}{H-d}\Big(\alpha_0 \frac{x^3}{6} + z_0 \frac{x^2}{2}\Big) + \gamma x + \delta,$$

$$P = -\frac{\lambda^2}{1-\varepsilon}\Big(\alpha_0 \frac{x^3}{6} + z_0 \frac{x^2}{2}\Big) + i\lambda \frac{H}{l}(\gamma x + \delta) - \alpha_0 x - z_0.$$

With allowance for (6) and the radiation condition, the potential outside the plate is written as

$$\Phi = \begin{cases} R\exp(i\lambda(x-1/2)), & x > 1/2. \\ T\exp(-i\lambda(x+1/2)), & x < -1/2. \end{cases}$$

Thus, for six unknown constants (α_0, z_0, γ, δ, R, and T) we have two conjugation conditions (8) at the plate edges and two equations of rigid-body motion of the plate:

$$-\omega^2 M l z_0 = \rho g l^2 \int_{-1/2}^{1/2} P\,dx, \qquad -\omega^2 I \alpha_0 = \rho g l^3 \int_{-1/2}^{1/2} Px\,dx$$

(M is the weight of the plate and I is the moment of inertia of the plate). Substituting the formula for P into these relations and integrating, we obtain

$$z_0\Big(1 + \frac{\lambda^2}{24(1-\varepsilon)} - \lambda^2 b\Big) - i\lambda \frac{H}{l}\delta = 0, \quad \alpha_0\Big(1 + \frac{\lambda^2}{40(1-\varepsilon)} - \lambda^2 b\Big) - i\lambda \frac{H}{l}\gamma = 0,$$

where $b = B/G$. Thus, we have a homogeneous system of sixth-order algebraic linear equations. The existence condition for a nontrivial solution is the equality to zero of the determinant of this system. By virtue of symmetry properties, we can distinguish between modes that are symmetric and antisymmetric about the plane $x = 0$. Then, the system is divided into two systems of the third order.

Thus, determination of eigenfrequencies reduces to seeking roots of the third-order polynomials:

$$[i/(3(1-\varepsilon)^2) + 4ib/(1-\varepsilon)]\lambda^3 - 2\lambda^2/(1-\varepsilon) - 4i\lambda/(1-\varepsilon) = 0$$

for symmetric modes, and

$$[i/(15(1-\varepsilon)^2) + 4ib/(1-\varepsilon)]\lambda^3 - [i/(5(1-\varepsilon)) + 8b]\lambda^2 - 4i\lambda/(1-\varepsilon) + 8 = 0$$

for antisymmetric modes.

For symmetric modes, we obtain one eigenvalue $\lambda_1 = 0$ and two complex eigenvalues, and for antisymmetric modes, we have one purely imaginary eigenvalue and two complex eigenvalues. All eigenvalues are located in the lower half-plane and are symmetric about the imaginary axis.

Flexible Plate. We consider flexible vibrations of the plate. We assume that the potential and displacements are periodic in y: $\varphi = \sqrt{gH}\, l\Phi(x)\exp(i(k_2 y - \omega t))$, where k_2 is the dimensionless wavenumber in the y direction and Φ satisfies the equation

$$\Big(\frac{d^2}{dx^2} - k_2^2\Big)^3 \Phi + (G - B\lambda^2)\Big(\frac{d^2}{dx^2} - k_2^2\Big)\Phi + \frac{G\lambda^2}{1-\varepsilon}\Phi = 0 \quad \text{for} \quad |x| < 1/2 \tag{9}$$

and Eq. (6) for $|x| > 1/2$. From Eq. (6) and the radiation conditions it follows that outside the plate, the solution is

$$\Phi = \begin{cases} R\exp[i\sqrt{\lambda^2 - k_2^2}(x - 1/2)], & x > 1/2, \\ T\exp[-i\sqrt{\lambda^2 - k_2^2}(x + 1/2)], & x < -1/2. \end{cases} \tag{10}$$

In this case, the branch of the root is selected so as to ensure the radiation condition, i.e., $-\pi/2 < \arg\sqrt{\lambda^2 - k_2^2} \leqslant \pi/2$. The radiation condition is used here to select a unique solution. For $\mathrm{Re}\sqrt{\lambda^2 - k_2^2} > 0$, the solution corresponds to waves that move away from the plate, and for $\mathrm{Re}\sqrt{\lambda^2 - k_2^2} = 0$, it is localized near the plate and is damped exponentially away from it.

The conjugation conditions have the form (8), and the boundary conditions (5) are

$$\Big(\frac{d^2}{dx^2} - \nu k_2^2\Big)\Big(\frac{d^2}{dx^2} - k_2^2\Big)\Phi = 0, \quad \frac{d}{dx}\Big(\frac{d^2}{dx^2} - (2-\nu)k_2^2\Big)\Big(\frac{d^2}{dx^2} - k_2^2\Big)\Phi = 0. \tag{11}$$

The general solution of Eq. (9) is sought as a series in functions of the form $\exp(\sigma x)$, where σ satisfies the equation

$$(\sigma^2 - k_2^2)^3 + (G - B\lambda^2)(\sigma^2 - k_2^2) + G\lambda^2/(1-\varepsilon) = 0. \tag{12}$$

We note that Eq. (12) is cubic in $\sigma^2 - k_2^2$, and, hence, it can be solved invoking the Cardano formulas. Generally, Eq. (12) has six different roots σ_i ($i = 1$–6), and the functions $\exp(\sigma_i x)$ form a fundamental system of solutions of Eq. (9). However, for some values of the parameter λ, Eq. (12) has multiple roots. Such values of λ can be found from the Cardano formulas. We consider the case $k_2 = 0$. Equation (12) becomes

$$\sigma^6 + (G - B\lambda^2)\sigma^2 + G\lambda^2/(1-\varepsilon) = 0. \tag{13}$$

Equation (13) has the double root $\sigma = 0$ for $\lambda = 0$. The values of λ for which Eq. (13) has nonzero multiple roots are given by the relation

$$\Big(\frac{G - B\lambda^2}{3}\Big)^3 + \Big(\frac{G\lambda^2}{2(1-\varepsilon)}\Big)^2 = 0. \tag{14}$$

Since $B \ll G$, we can set $B = 0$ as a first approximation. Then, four roots are given by the relation

$$\lambda^4 = -4G(1-\varepsilon)^2/27. \tag{15}$$

The corresponding values of λ lie on the bisectrices of the quadrants. For $B \neq 0$, the correction to these values can be found by the perturbation method. The two other roots are sought by the asymptotic method. We replace $\lambda = G^{1/4}æ$ and write Eq. (14) in the form

$$(1 - \delta^2 æ^2)^3 + 27æ^4/(4(1-\varepsilon)^2) = 0, \qquad \delta^2 = B/\sqrt{G}. \tag{16}$$

In the limit $\delta \to 0$, the four roots remain finite and are defined by relation (15), and the two other roots are not limited. We seek them in the form $æ = c/f(\delta)$, where $f(\delta) \to 0$ as $\delta \to 0$ and c is a constant. Substituting this expression into (16), we obtain

$$1 - 3\frac{\delta^2}{f^2}c^2 + 3\frac{\delta^4}{f^4}c^4 - \frac{\delta^6}{f^6}c^6 + \frac{27}{4(1-\varepsilon)^2}\frac{c^4}{f^4} = 0. \tag{17}$$

As $\delta \to 0$, the main terms of the equations should have the same order, and, hence, $\delta^6/f^6 = 1/f^4$ and $f = \delta^3$. Equation (17) is written as

$$\delta^{12} - 3\delta^8 c^2 + 3\delta^4 c^4 - c^6 + 27c^4/(4(1-\varepsilon)^2) = 0.$$

In the limit $\delta \to 0$, we have $c^4[c^2 - 27/(4(1-\varepsilon)^2)] = 0$, and, hence, we obtain two nonzero values of c which correspond to the values $\lambda = \pm 3\sqrt{3}/(2(1-\varepsilon))(G/B^{3/2})$.

In the case $k_2 \neq 0$, the values of λ for which nonzero multiple solutions of Eq. (12) exists are the same as in the case $k_2 = 0$. Equation (12) has zero multiple roots $\sigma_{1,2} = 0$ for $\lambda_0^2 = (k_2^6 + Gk_2^2)/(Bk_2^2 + G/(1-\varepsilon))$. Both values $\pm\lambda_0$ lie on the real axis.

For all values of λ, except for those mentioned above, the solutions $\exp(\sigma_i x)$ ($i = 1$–6) form a fundamental system of solutions of Eq. (9) for $k_2 = 0$. Under the plate, the liquid-velocity potential is $\Phi = \sum_{k=1}^{6} C_k \exp(\sigma_k x)$ $(-1/2 < x < 1/2)$ and outside the plate, it has the form (10).

From the conjugation conditions and the boundary conditions, we obtain a system of linear algebraic equations whose determinant should be equal to zero. The determinant of the system is an analytic function

TABLE 1

ε	λ			
	Symmetric modes		Antisymmetric modes	
0	0;	$\pm 1.73 - 3.0i$	$-4.64i$;	$\pm 3.51 - 3.67i$
0.01	0;	$\pm 1.75 - 2.97i$	$-4.476i$;	$\pm 3.55 - 3.69i$
0.08	0;	$\pm 1.85 - 2.75i$	$-3.53i$;	$\pm 3.82 - 3.74i$
0.1	0;	$\pm 1.87 - 2.7i$	$-3.34i$;	$\pm 3.89 - 3.73i$

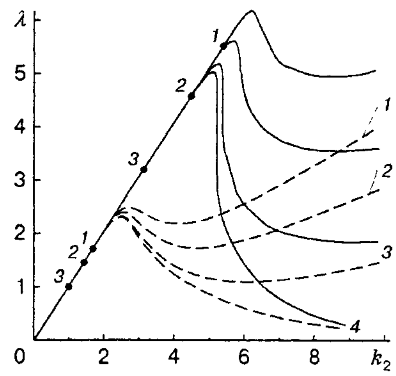

Fig. 1

of λ over the entire complex plane, except for singular points and the cut on the real axis $[-k_2, k_2]$. Singular points can be branch points of the roots, i.e., values of the spectral parameter λ for which Eq. (12) has multiple roots. In this case, the system of particular solutions $\exp(\sigma_i x)$ becomes linearly dependent and one of the solutions $\exp(\sigma_i x)$ should be replaced by $x \exp(\sigma_i x)$.

By virtue of symmetry of the problem, the eigenvibrations of the system are conveniently divided into symmetric and antisymmetric modes about the plane $x = 0$. For symmetrical modes, the velocity potential under the plate is written as $\Phi = \sum\limits_{k=1}^{3} c_k \cosh(\sigma_k x)$ ($|x| < 1/2$) and for antisymmetric modes, $\Phi = \sum\limits_{k=1}^{3} c_k \cosh(\sigma_k x)$ ($|x| < 1/2$).

From the boundary conditions and the conjugation conditions, we obtain two systems of linear algebraic equations of the fourth order. Outside the interval $[-k_2, k_2]$, the eigenvalues are sought numerically using the argument principle [5]. The parameter k_2 is specified, and the dependence of the dimensionless frequency λ on k_2 for the eigenmodes is examined. It should be noted that the eigenvalues are symmetric about the imaginary axis. If λ is an eigenvalue, $-\bar{\lambda}$ is also an eigenvalue.

At the edges of the cut $[-k_2, k_2]$, the determinant of the system is a purely imaginary value. The determinant of the system was calculated for various values $\lambda \in [-k_2, k_2]$. The change of sign of the imaginary part indicates the presence of zero.

Numerical Results. Numerical calculations were performed for a model of a floating pontoon airport. for the following parameters of the problem [1]: width of the plate $l = 1250$ m, thickness $h = 4.5$ m, draught $d = 1.8$ m, water layer depth $H = 22$ m, weight of the plate per unit area $m = 1.8$ tons/m^2, Poisson's constant $\nu = 0.3$, and rigidity $D = 10^{12}$ and 10^{14} N/m. The dimensionless input parameters are $b = B/G = 0.000025$, $\varepsilon = 0.08$, and $G = 10000$ and 100.

From the calculations it follows that for a flexible plate, as for a rigid plate, there are six eigenmodes: three symmetric and three antisymmetric modes. For $k_2 = 0$ and $D \to \infty$, the hydroelastic frequencies tend to the corresponding values for a rigid plate.

The calculations show that the small parameter $\varepsilon = d/H$ has a significant effect on the eigenvalues. Indeed, in the derivation of the shallow-water equation, expansion is performed in the small parameter H^2/l^2, which for platforms of this type is about 0.0003. In this case, $\varepsilon = 0.08$. The parameter ε is large compared to H^2/l^2, and, hence, it should be taken into account for platforms of this type.

Table 1 shows the eigenvalues (dimensionless) for an absolutely rigid plate for various ε. It is obvious that the effect of ε is most pronounced for the first antisymmetric mode.

The effect of the parameter ε on the eigenvalues of a flexible plate is not only quantitative but also qualitative. For $\varepsilon \neq 0$, the first symmetric mode in a certain interval of values k_2 is waveguide. For each

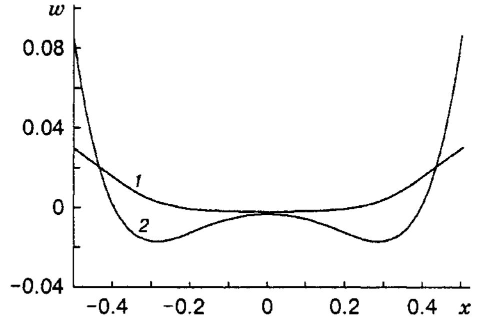

Fig. 2

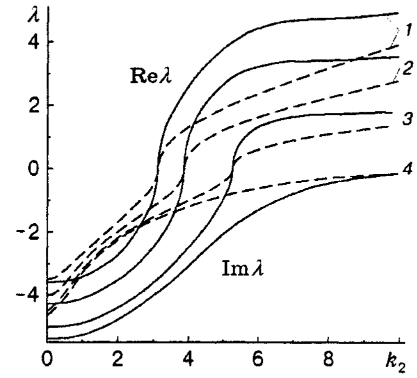

Fig. 3

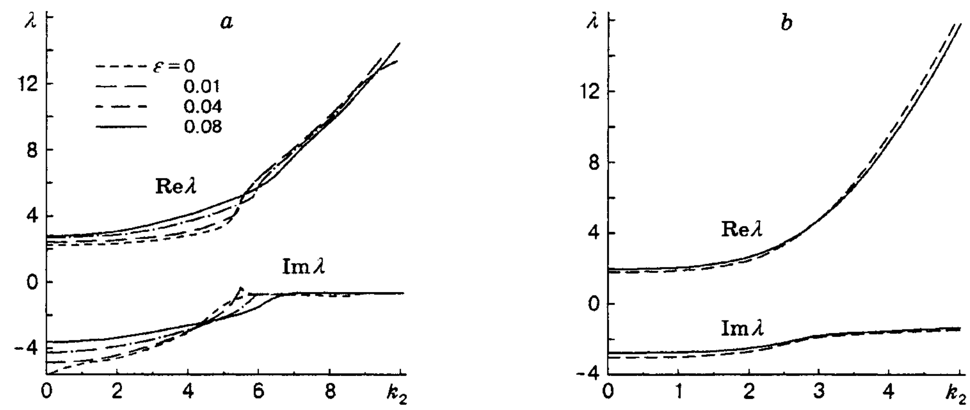

Fig. 4

ε, a certain critical value $k_2^*(\varepsilon)$ exists such that the first symmetric mode ceases to be waveguide mode and becomes outgoing and growing at infinity. For $k_2 > k_2^*$ there are no waveguide modes. As $\varepsilon \to 0$, the critical point tends to zero. Figure 1 shows the reduced frequency λ of the first symmetric mode versus the wavenumber k_2 for various values of ε (curves 1–4 refer to $\varepsilon = 0.08$, 0.04, 0.01, and 0, respectively) and $G = 10{,}000$ and 100 (solid and dashed curves, respectively). The points show the critical values for which the waveguide mode becomes outgoing. For $k_2 > k_2^*$, the eigenmode is exponentially growing at infinity with coefficient $k_1 = \sqrt{k_2^2 - \lambda^2}$. The vibration form of the plate for the waveguide mode is shown in Fig. 2 (curve 1 refers to $k_2 = 1$ and curve 2 refers to $k_2 = 4$) for $G = 10{,}000$ and $\varepsilon = 0.08$.

The first antisymmetric mode also has a certain critical value $k_2^0(\varepsilon)$. For $k_2 < k_2^0(\varepsilon)$, the eigenvalue λ is purely imaginary, $\lambda \to 0$ as $k_2 \to k_2^0(\varepsilon)$, and for $k_2 > k_2^0(\varepsilon)$, λ becomes a real quantity. Figure 3 shows the eigenvalue λ versus k_2 for the first antisymmetric mode for various values of ε (curves 1–4 refer to $\varepsilon = 0.08$, 0.04, 0.01, and 0) and $G = 10{,}000$ and 100 (solid and dashed curves, respectively). The eigenmode is exponentially growing in x for all k_2.

The second modes (symmetric and antisymmetric) have complex eigenvalues. The eigenvalue λ versus k_2 for the second symmetric mode for various values of ε are given in Fig. 4a and b for $G = 10{,}000$ and 100, respectively. Figure 5 shows the eigenvalue λ versus k_2 for the second antisymmetric mode for various values of ε and $G = 10{,}000$. For these modes, the effect of the parameter ε becomes less pronounced and only quantitative. Qualitatively, the behavior of the curves for different values of the parameter ε is similar. However, for some combinations of the input parameters (for example, $G = 10{,}000$ and $\varepsilon = 0.01$) there is a sharp decrease in the damping coefficient $\operatorname{Im}\lambda$ for $k_2 = 5.5$ for the second symmetric mode. The eigenfrequency of the second antisymmetric mode practically does not depend on ε.

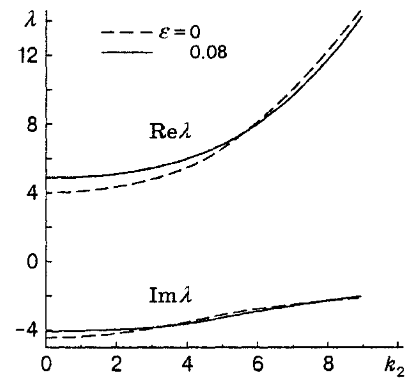

Fig. 5

It should be noted that the plate eigenvibrations found cannot be excited by incoming waves since they are described by different dispersion relations. For the eigenvibrations of the plate, the wavenumber k_1 in the x direction is purely imaginary. Since $\lambda^2 = k_1^2 + k_2^2$, we have $\lambda \leqslant k_2$ for the eigenvibrations and $\lambda \geqslant k_2$ for waves on water. The single case where the dispersion relation for the eigenvibrations corresponds to waves on water is the critical point for the first symmetric mode, at which $k_1 = 0$ and $k_2 = \lambda$. This case corresponds to a wave propagating along the plate in the absence of reflected and transmitted waves. The model of an infinitely long plate becomes inapplicable since the number of equations is larger than the number of unknowns. A solution can be constructed only when the wave has a wavenumber and a frequency corresponding to the critical point. The system then becomes compatible.

Behavior of the Plate under External Loading. We consider the case where the plate is subjected to periodic external load $q_0(y)\exp(-i\omega t)$ that does not depend on x:

$$q_0(y) = \begin{cases} q_0, & |y| < y_0, \\ 0, & |y| > y_0. \end{cases}$$

The plate performs forced vibrations under the same harmonic law $w(x, y)\exp(-i\omega t)$. The function $w(x, y)$ is a solution of the equation

$$i\lambda w(x,\ y) = \frac{H}{l}(1-\varepsilon)\Delta\Phi, \quad \Delta^3\Phi + G(1-b\lambda^2)\Delta\Phi + \frac{G\lambda^2}{1-\varepsilon}\Phi = i\frac{G\lambda}{1-\varepsilon}\frac{q_0(y)}{\rho g h}. \tag{18}$$

We use a Fourier transform along the y coordinate:

$$\hat{q}_0(k_2) = \frac{1}{\sqrt{2\pi}}\int\limits_{-y_0}^{y_0}\exp(-ik_2 y)q_0\,dy = \sqrt{\frac{2}{\pi}}\frac{q_0\sin(k_2 y_0)}{k_2},$$

$$\hat{\Phi}(x,\ k_2) = \frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}\exp(-ik_2 y)\Phi(x, y)\,dy.$$

Assuming that as $|y| \to \infty$, $\Phi(x, y)$ tends to zero together with derivatives with respect to y up to the fifth order, inclusively, we obtain

$$\Big[\Big(\frac{d^2}{dx^2} - k_2^2\Big)^3 + G(1-b\lambda^2)\Big(\frac{d^2}{dx^2} - k_2^2\Big) + \frac{G\lambda^2}{1-\varepsilon}\Big]\hat{\Phi}(x, k_2) = i\frac{G\lambda}{1-\varepsilon}\frac{\hat{q}_0(k_2)}{\rho g h}. \tag{19}$$

The general solution of this equation is $\hat{\Phi}(x, k_2) = \sum_{j=1}^{3} a_j\cosh(\sigma_j x) + \hat{\Phi}_0(x, k_2)$, where σ_j are solutions of Eq. (12), a_j are arbitrary constants, and $\hat{\Phi}_0(x, k_2)$ is a particular solution of Eq. (19). The particular solution has the form

$$\hat{\Phi}_0(x,k_2) = -i\,\frac{\hat{q}_0(k_2)}{\rho g h(1-\varepsilon)}\,\frac{G\lambda}{k_2^6 + G(1-b\lambda^2)k_2^2 - G\lambda^2/(1-\varepsilon)},$$

if the denominator is different from zero, and, otherwise,

$$\hat{\Phi}_0(x,k_2) = i\,\frac{\hat{q}_0(k_2)}{\rho g h(1-\varepsilon)}\,\frac{G\lambda}{3k_2^4 + G(1-b\lambda^2)}\,\frac{x^2}{2}.$$

From (18) we obtain

$$\hat{w} = i\,\frac{H(1-\varepsilon)}{l\lambda}\Big[\sum_{j=1}^{3} a_j(\sigma_j^2 - k_2^2)\cosh(\sigma_j x) + \Big(\frac{d^2}{dx^2} - k_2^2\Big)\hat{\Phi}_0(x,k_2)\Big]. \qquad (20)$$

Determining the coefficients a_j from boundary conditions (11) and the conjugation conditions, we have a system of linear algebraic equations whose eigenvibrations were studied above. According to the Cramer rule, the solution of this system for the coefficients a_j has the form $a_j = A_j/D$, where D is the determinant of the system. From (20), we obtain

$$\hat{w} = i\,\frac{H(1-\varepsilon)}{l\lambda}\Big[\sum_{j=1}^{3} \frac{A_j}{D}(\sigma_j^2 - k_2^2)\cosh(\sigma_j x) + \Big(\frac{d^2}{dx^2} - k_2^2\Big)\hat{\Phi}_0(x,k_2)\Big].$$

Using an inverse Fourier transform, we find $w(x,y)$. As $y \to \infty$, the integral can be calculated by means of residues. The residues on the real axis are of primary interest, and the remaining residues give a solution that decays in the y direction. If the frequency of the external pressure field is in the range of existence of the waveguide mode, the local periodic load generates a wave of the form $C(x)\exp(ik_2 y)$, which propagates along the plate practically without damping, and can set the entire plate in vibration.

Schulkes and Sneyd [6] and Marchenko [7] showed that resonance is observed in the case of a moving periodic load if the external pressure field moves with the group velocity of the waveguide eigenmode, and the frequency corresponds to the eigenfrequency of the waveguide mode in a coordinate system attached to the moving load. In this case, for $t \to \infty$, the solution is proportional to $\exp(i\omega t)\sqrt{t}$. For a flexible plate, the resonant velocity is equal to $\sqrt{gH}$. Thus, in designing a floating airport on shallow water, it is necessary to take measures to suppress the waveguide mode.

The work was performed within the framework of the integration project No. 43 of the Russian Academy of Sciences and supported by the Russian Foundation for Fundamental Research (Grant No. 97- 01-00897).

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